1 Review, Limits, and Continuity

1.1 Functions

Definition 1.1 (Function)**.** - a function is a rule that associates to each x-value one y-value.

Common functions you'll see are

1. $y = \sqrt{x-1}$

2. $y = x^2$

3. $y = \dfrac{1}{x+4}$

Since we're on the topic of functions, here's an example of one that isn't a function.

$$y^2 = x$$

Why?

Remember, every function associates an x-value to **ONE** y-value. Another way to think about it is

If two different y-values can come from the ***same*** *x-value, it's not a function.*

x	y
4	-2
4	2

See what happened? One x-value, two y-values. That's bad. So this isn't a function. Let's move on!

Domain Issues

Let's stop for a moment and look at the domain. In this course, domain problems don't come up too much. We focus a lot on functions that 'work.' But the first week or so, we'll be looking at piece-wise functions and these functions have domain issues all over the place.

Definition 1.2 (Domain)**.** This is the collection of all *legal* inputs. If you can plug in a number and get a 'number' out, then you're in the domain.

I always think of the domain as stuff that doesn't ***break*** *the function.*

So what are some common domain issues?

1. Square rooting a negative number: $y = \sqrt{x-1}$
   Setting $x - 1 \geq 0$ gives us a domain of $x \geq 1$. Plug any number less than 1 and you square root a negative.

2. Dividing by 0

   Examples:

   (a) $y = \dfrac{1}{x}$

   (b) $y = \dfrac{x}{x^2 - 1}$

(c) $y = \dfrac{x}{\sqrt{x-2}-4}$

Of course, dividing by 0 is the worst one. But you'll see that dividing by 0 shows up quite a bit in this course.

Solution to 2c

You got two things to worry about.

1. When does the denominator equal 0

2. When are we square rooting a negative

Ok, let's get started.

1. Set $\sqrt{x-2}-4=0$.

$$\begin{aligned}\sqrt{x-2}-4&=0\\ \rightarrow \sqrt{x-2}&=4\\ \rightarrow x-2&=16\\ \rightarrow x&=18\end{aligned}$$

2. Set $\sqrt{x-2} \geq 0$

$$\begin{aligned}x-2&\geq 0\\ x&\geq 2\end{aligned}$$

Easy, right?

So the overall domain would look something like this:

$$\text{Domain: } [2, 18) \cup (18, \infty)$$